

Question Paper

Quantitative Methods – I (131) : January 2004

Section A : Basic Concepts (40 Marks)

- • This section consists of questions with serial number 1 - 40.
- • Answer all questions.
- • Each question carries one mark.

1. If all the terms of an arithmetic progression are multiplied by a constant quantity the resulting terms will always form [< Answer >](#)
- a. A geometric progression
 - b. A harmonic progression
 - c. An arithmetic progression
 - d. Either a geometric progression or a harmonic progression
 - e. Either a geometric progression or an arithmetic progression.
2. If the first term in a geometric progression is greater than 1 and the common ratio is less than 1, then the consecutive terms will be [< Answer >](#)
- a. The consecutive terms will be in increasing order
 - b. The consecutive terms will be in decreasing order
 - c. The consecutive terms will be same
 - d. All the consecutive terms will be less than 1
 - e. All the consecutive terms will be greater than 1.
3. The reciprocals of the terms in a harmonic progression are [< Answer >](#)
- | | |
|--------------------------------|-------------------------------|
| a. In geometric progression | b. In harmonic progression |
| c. In arithmetic progression | d. Always in decreasing order |
| e. Always in increasing order. | |
4. The set of whole numbers includes the set of [< Answer >](#)
- | | |
|------------------------|---------------------|
| a. Natural numbers | b. Rational numbers |
| c. Negative numbers | d. Complex numbers |
| e. Irrational numbers. | |
5. According to the inverse property of addition [< Answer >](#)
- a. For every real number there exists another real number such that the sum of the two real numbers is equal to 1
 - b. For every real number there exists another number such that the sum of the two real numbers is equal to 0
 - c. The addition of zero to any real number is equal to that real number
 - d. For every real number there exists another real number such that the product of the two numbers is equal to 1
 - e. The product of any real number with 1 is equal to that real number.
6. Which of the following is **not** an unbounded interval? [< Answer >](#)
- a. The set of all real numbers greater than a specified real number
 - b. The set of all real numbers less than a specified real number
 - c. The set of all real numbers greater than or equal to a specified real number
 - d. The set of all real numbers less than or equal to a specified real number
 - e. The set of all real numbers greater than one real number and less than another real number.
7. If $X = p^m$ and $Y = p^n$, then the product of X and Y is equal to [< Answer >](#)
- | | | | | |
|-----------------|------------|--------------|--------------|--------------|
| a. $m \times n$ | b. $m + n$ | c. p^{m+n} | d. p^{m-n} | e. $p^{m/n}$ |
|-----------------|------------|--------------|--------------|--------------|

8. The logarithm of a number

[< Answer >](#)

- a. Is always expressed with respect to base 10
- b. Is always expressed with respect to base 1
- c. Is always expressed with respect to base e
- d. Is equal to the base which must be raised to a given exponent in order to get the number
- e. Is equal to the exponent to which a given base must be raised in order to get the number.

9. A function is a rule or correspondence which always associates to

[< Answer >](#)

- a. Each number x in a set A a unique number $f(x)$ in a set B
- b. All the numbers in a set A the same number in a set B
- c. All the numbers in a set A a fixed positive number
- d. All the numbers in a set A a fixed negative number
- e. Each number x in a set A , two numbers in a set B .

10. In a two dimensional plane the graph of a function of the form $y = a + bx$ is a

[< Answer >](#)

- a. Circle
- b. Curve
- c. Square
- d. Rectangle
- e. Straight line.

11. The accumulated amount at the end of t years, of a given amount of investment, under compound interest, is an example of

[< Answer >](#)

- a. Linear function in one independent variable
- b. Linear function in two independent variables
- c. Logarithmic function
- d. Exponential function
- e. Inverse function.

12. For any function $f(x)$ the limit of $f(x)$ as x approaches a value a is

[< Answer >](#)

- a. Always equal to a
- b. Always a value greater than a
- c. Always a value less than a
- d. Always less than $f(a)$
- e. A value L , such that $f(x)$ approaches L as x approaches a .

13. The derivative of a function $f(x)$ at $x = a$ is

[< Answer >](#)

- a. Always equal to $f(a)$
- b. Always equal to a
- c. The rate of change in the value of the function at $x = a$
- d. The limit of $f(x)$ as x approaches a
- e. The ratio of $f(a)$ to a .

14. If the derivative of a function $f(x)$ is zero for all values of x , then it implies that

[< Answer >](#)

- a. $f(x)$ has a constant value for all values of x
- b. $f(x)$ increases at a constant rate as x increases
- c. $f(x)$ decreases at a constant rate as x increases
- d. $f(x)$ increases at a variable rate as x increases
- e. $f(x)$ decreases at a variable rate as x increases.

15. For a function $f(x)$ the first derivative is positive and the second derivative is negative at $x = a$. This means that

[< Answer >](#)

- a. $f(x)$ is minimum at $x = a$
- b. $f(x)$ is maximum at $x = a$
- c. $f(x)$ is increasing at an increasing rate at $x = a$
- d. $f(x)$ is increasing at a decreasing rate at $x = a$
- e. $f(x)$ is decreasing at an increasing rate at $x = a$.

16. A function $f(x)$ is said to be monotonically increasing if

[< Answer >](#)

- a. The first derivative of $f(x)$ is a constant for all values of x

- b. The first derivative of $f(x)$ is negative for all values of x
- c. The first derivative of $f(x)$ is positive for all values of x
- d. The first derivative of $f(x)$ is zero for all values of x
- e. The first derivative of $f(x)$ is equal to 1 for all values of x .

[< Answer >](#)

17. If the first derivative of a function $f(x)$ is zero at $x = a$ then

- a. $f(x)$ will be maximum at $x = a$
- b. $f(x)$ will be minimum at $x = a$
- c. $f(x)$ will be zero at $x = a$
- d. $f(x)$ will be maximum at $x = a$ if the second derivative is negative at $x = a$
- e. $f(x)$ will be minimum at $x = a$ if the second derivative is negative at $x = a$.

[< Answer >](#)

18. Which of the following is **true**?

- a. In a logarithmic function the logarithmic operation is applied on the dependent variable
- b. In a logarithmic function the base of the logarithm is always 10
- c. In a logarithmic function the independent variable cannot be negative
- d. In a logarithmic function the base of the logarithm can be any real number
- e. None of the above.

[< Answer >](#)

19. Three simultaneous equations involving three variables can be solved if the three equations

- a. Can be satisfied by the three variables assuming equal value in all the equations
- b. Can be satisfied when two out of the three variables assume equal value in all the equations
- c. Can be satisfied when each of the three variables assumes a different value in each of the three equations
- d. Can be satisfied when each of the variables are set to zero in all the equations
- e. Can be satisfied when the three variables respectively assume the same values in the three equations.

[< Answer >](#)

20. In which of the following conditions the technique of linear programming **need not** be applied for decision making?

- a. There is an objective to be achieved
- b. The objective can be expressed in the form of a linear function
- c. There are alternative courses of action
- d. There is no constraint in the supply of the resources
- e. There are multiple decision variables to which resources must be allocated.

[< Answer >](#)

21. Which of the following is **false** with regard to linear programming problems?

- a. The contribution of each unit of the decision variables towards the objective to be achieved is known
- b. The consumption of resources by each unit of the decision variables is known
- c. The values of the decision variables in the optimal solution will always be whole numbers
- d. Linear programming is used either to maximize or to minimize the value of the objective function
- e. The optimal solution to any linear programming problem is one of the possible feasible solutions.

[< Answer >](#)

22. In a linear programming problem, the limitations on the use of resources are represented by the

- a. Objective function
- b. Structural constraints
- c. Nonnegativity constraints
- d. Decision variables
- e. Solutions.

[< Answer >](#)

23. In the graphical method of solving linear programming problems the feasible region is the set of all points

- a. Which do not satisfy any of the constraints
- b. Which satisfy exactly one of the constraints
- c. Which satisfy all the constraints
- d. At which the objective function has the same value

- e. At which the objective function is equal to zero. [< Answer >](#)
24. Which of the following is **false** with regard to the graphical method of solving linear programming problems? [< Answer >](#)
- It is applicable when there are two decision variables
 - The decision variables are represented by the horizontal and vertical axes
 - Straight lines are used to demarcate the feasible region
 - The feasible region shows the solutions that satisfy all the constraints
 - One of the corner points of the feasible region will always be at the origin.
25. In the graphical method of solving linear programming problems if there is a unique optimal solution, then the optimal solution [< Answer >](#)
- Is always found at the center of the feasible region
 - Is always at the origin
 - Lies outside the feasible region
 - Is located at one of the corner points of the feasible region
 - Always lies on one of the two axes.
26. Which of the following is **true** with regard to the simplex method of solving linear programming problems? [< Answer >](#)
- This method cannot be applied for solving problems involving two decision variables
 - At the optimal solution all the decision variables assume zero values
 - Slack variables are used to convert 'greater than or equal to' type constraints into equations
 - At the optimal solution all the slack variables assume zero values
 - The $Z_j - C_j$ value at the bottom of the solution column shows the value of the objective function for any solution.
27. Which of the following is **false** with regard to the simplex method of solving linear programming problems? [< Answer >](#)
- It involves an iterative procedure for arriving at the optimal solution
 - Slack variables are used to represent the unused resources
 - Slack variables make zero contribution towards the objective to be achieved
 - The $Z_j - C_j$ values indicate the variable to leave solution
 - The $Z_j - C_j$ values indicate whether the solution is optimal or not.
28. Which of the following is **false** with regard to histograms? [< Answer >](#)
- The class intervals are represented by the base of the rectangles
 - The frequencies are represented by the heights of the rectangles
 - If the class intervals are of equal width then the bases of the rectangles will be equal in length
 - The tallest rectangle in a histogram represents the class interval with the lowest frequency
 - None of the above.
29. Which of the following is meant by the skewness of a distribution? [< Answer >](#)
- The absence of symmetry in the distribution
 - The consistency of the data in the distribution
 - The dispersion of the data in the distribution
 - The central tendency of the distribution
 - None of the above.
30. In a positively skewed distribution [< Answer >](#)
- Majority of the observations are concentrated towards the higher end of the scale
 - Majority of the observations are concentrated towards the lower end of the scale
 - Majority of the observations are concentrated at the center of the distribution
 - The observations have the same frequency
 - The distribution of the data is symmetrical.
31. Which of the following measures represents the scatter of the values in a data set? [< Answer >](#)

- a. Arithmetic mean b. Geometric mean
 - c. Harmonic mean d. Median e. Standard deviation.
32. Which of the following measures is based only on two observations in a data set?
- a. Arithmetic mean b. Harmonic mean c. Range
 - d. Mean absolute deviation e. Standard deviation.
33. Which of the following measures is **not** affected by the presence of extreme values in a data set?
- a. Range b. Arithmetic mean c. Standard deviation
 - d. Variance e. Median.
34. Which of the following measures will remain unchanged when every observation in the data set is divided by the same quantity?
- a. Range b. Quartile deviation
 - c. Standard deviation d. Coefficient of variation
 - e. Mode.
35. If a constant quantity is subtracted from every observation in a data set then the range of the resulting set of values will be equal to
- a. Range of the original data set plus the constant quantity
 - b. Range of the original data set minus the constant quantity
 - c. Range of the original data set
 - d. Range of the original data set multiplied by the constant quantity
 - e. Range of the original data set divided by the constant quantity.
36. If every observation in a data set is increased by a constant quantity then the coefficient of variation of the resulting set of values will be
- a. Less than the coefficient of variation of the original data set
 - b. Greater than the coefficient of variation of the original data set
 - c. Equal to the coefficient of variation of the original data set
 - d. Equal to the coefficient of variation of the original data set plus the square root of the constant quantity
 - e. Equal to the coefficient of variation of the original data set multiplied by the square root of the constant quantity.
37. Two events **A** and **B** are such that their joint probability is equal to the product of their marginal probabilities. Which of the following best describes events **A** and **B**?
- a. **A** and **B** are dependent events
 - b. **A** and **B** are independent events
 - c. **A** and **B** are mutually exclusive events
 - d. **A** and **B** are collectively exhaustive events
 - e. None of the above.
38. If two events **A** and **B** are independent then, the conditional probability of event **A** given that event **B** has occurred, is equal to
- a. Joint probability of events **A** and **B**
 - b. Conditional probability of event **B** given event **A**
 - c. Marginal probability of event **B**
 - d. Marginal probability of event **A**
 - e. Zero.
39. Which of the following are the conditions for applying the Baye's theorem for computing posterior probabilities of certain events?
- a. The events must be non-mutually exclusive
 - b. The events must be mutually exclusive
 - c. The events must not be collectively exhaustive

- d. The events must be collectively exhaustive
- e. Both (b) and (d) above.

40. Which of the following statements is most appropriate if certain events are mutually exclusive and collectively exhaustive?

[< Answer >](#)

- a. Each of the events has a zero probability
- b. Some of the events will definitely have zero probability
- c. The sum of the probabilities of the events will be equal to 1
- d. The sum of the probabilities of the events will be less than 1
- e. The sum of the probabilities of the events will be more than 1.

END OF SECTION A

Section B : Problems (60 Marks)

- • This section consists of questions with serial number 41 - 79.
- • Answer all questions.
- • Marks are indicated against each question.

41. A data set contains the following observations:

10, 12, 8, 14, 15, 7, 17, 20

What is the range of the data set?

- a. 8 b. 9 c. 10 d. 12 e. 13

(1 mark)

[< Answer >](#)

42. A data set contains the following observations:

3, 8, 5, 6, 10, 7

What is the arithmetic mean of the data set?

- a. 3.5 b. 4.5 c. 5.5 d. 6.5 e. 7.5

(1 mark)

[< Answer >](#)

43. A data set contains the following observations:

7, 8, 6, 11, 9, 5, 7, 6, 4, 1, 5, 7, 2, 7

What is mode of the data set?

- a. 4 b. 5 c. 6 d. 7 e. 11

(1 mark)

[< Answer >](#)

44. The coefficient of variation of a data set is 20% and the variance of the data set is 16. What is the mean of the data set?

- a. 4 b. 20 c. 25 d. 80 e. 125

(1 mark)

[< Answer >](#)

45. The variance of a data set is 9. If every item in the data set is multiplied by 2 then, what is the variance of the resulting data set?

- a. 9 b. 12 c. 36 d. 72 e. 324

(1 mark)

[< Answer >](#)

46. The sum of the observations in a data set containing 10 observations is 120 and the standard deviation of the data set is 3. If 3 is added to every observation in the data set then, what will be the coefficient of variation of the resulting data set?

- a. 0 b. 7.5% c. 20% d. 25% e. 50%

(1 mark)

[< Answer >](#)

47. The following details are available with regard to a sample of ten observations:

Arithmetic mean of the first four observations	=	16
Arithmetic mean of the next six observations	=	11

[< Answer >](#)

$$\sum_{i=1}^{10} x_i^2 = 1726$$

What is the standard deviation of the sample?

- a. 2 b. 3 c. 4 d. 5 e. 6

(1 mark)

[< Answer >](#)

48. In a textile factory there are 50 skilled workers, 125 semiskilled workers and 75 unskilled workers. It has been observed that on average a unit length of a particular fabric is woven by a skilled worker in 4 hours, by a semiskilled worker in 5 hours and by an unskilled worker in 6 hours. After three years of experience the semiskilled workers are expected to become skilled and the unskilled workers to become semiskilled. It is assumed that there will be no turnover of the workers within the next three years. What will be the change in time taken for weaving an unit length of the same fabric, after three years?

- a. No change b. Reduction by 1.60 hour
c. Reduction by 0.80 hour d. Increase by 0.80 hour
e. Increase by 1.60 hours.

(1 mark)

[< Answer >](#)

49. A group of salesmen from the same industry consists of some sales men who have 5 years of experience and others who have 10 years of experience. Sixty percent of the salesmen in the group have 5 years of experience and their average salary is Rs.7,000 per month. The average salary for the entire group is Rs.9,000.

What is the average salary of the salesmen who have 10 years of experience?

- a. Rs.4,200 b. Rs.4,800 c. Rs.8,000 d. Rs.10,000
e. Rs.12,000

(1 mark)

[< Answer >](#)

50. For a class consisting of 60 students, a test on Mathematics was conducted. The marks obtained by the students are given below:

Marks obtained out of 120	Numbers of students
0 - 20	6
20 - 40	14
40 - 60	18
60 - 80	12
80 - 100	8
100 - 120	2

What is the median marks obtained by the students in the class?

- a. 35.5 b. 45.8 c. 50.6 d. 55.4 e. 65.4

(2 marks)

[< Answer >](#)

51. The equation of a straight line is $\frac{y}{3} + \frac{2}{5}x = 2$

Which of the following statements is/are **correct**?

- a. The intercept on the Y axis is 5 b. The intercept on the Y axis is 6
c. The straight line has a positive slope d. The straight line has a negative slope
e. Both (b) and (d) above.

(1 mark)

52. The terms in a series are as given below:

3, 6, 12, 24, 48.....

The eighth term of the series is

- a. 72 b. 96 c. 192 d. 384 e. 768

(1 mark)

[< Answer >](#)

53. $\log_2 8 + \log_3 27 =$

- a. 3 b. 4 c. 5 d. 6 e. 8

(1 mark)

[< Answer >](#)

54. The first term of an arithmetic progression (**A.P**) is 6 and the last term of the **A.P** is 34. If there are eight terms in the **A.P.**, then what is the sum of the **A.P.**?

- a. 144 b. 148 c. 152 d. 156 e. 160

(1 mark)

[< Answer >](#)

55. $\lim_{t \rightarrow 0} \frac{2 - \sqrt{4 - t^2}}{t^2} =$

- a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. 2 d. 4 e. $\sqrt{2}$

(1 mark)

[< Answer >](#)

56. $u = \ln \left(t + \sqrt{t^2 + 4} \right)$
 $\frac{du}{dt} =$

- a. $t + \sqrt{t^2 + 4}$ b. $\sqrt{t^2 + 4}$ c. $(t^2 + 4)^{\frac{-1}{2}}$ d. $(t^2 + 4)^{-1}$ e. $t^2 + 4$

(2 marks)

[< Answer >](#)

57. $\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 2}{3x^2 - 4x} =$

- a. 0 b. $\frac{1}{3}$ c. $\frac{1}{2}$ d. $\frac{2}{3}$ e. ∞

(1 mark)

[< Answer >](#)

58. $u = 2t^3 - 21t^2 + 36t + 50$

Which of the following statements is **correct**?

- a. **u** is minimum at **t** = 1 and maximum at **t** = 6
b. **u** is minimum at **t** = 2 and maximum at **t** = 5
c. **u** is minimum at **t** = 4 and maximum at **t** = 3
d. **u** is minimum at **t** = 5 and maximum at **t** = 4
e. **u** is minimum at **t** = 6 and maximum at **t** = 1.

(2 marks)

[< Answer >](#)

59. $\int_0^1 2xe^{x^2+5} dx =$ [< Answer >](#)

- a. e b. e^6 c. $e^6(e-1)$ d. e^5 e. $e^5(e-1)$
(1 mark)

60. $\int_1^2 \frac{3x^2}{x^3+4} dx =$ [< Answer >](#)

- a. $\ln 60$ b. $\frac{\ln 5}{\ln 12}$ c. $\frac{\ln 12}{\ln 5}$ d. $\ln\left(\frac{12}{5}\right)$ e. $\ln\left(\frac{5}{12}\right)$
(1 mark)

61. **A**, **B** and **C** are mutually exclusive and collectively exhaustive events, resulting from an experiment. **B** is twice as likely as **A**, and **C** is 2.5 times as likely as **B**. [< Answer >](#)

Which of the following is **correct**?

- a. $P(A) = 3\%$, $P(B) = 6\%$, $P(C) = 15\%$
b. $P(A) = 5\%$, $P(B) = 10\%$, $P(C) = 25\%$
c. $P(A) = 8\%$, $P(B) = 16\%$, $P(C) = 40\%$
d. $P(A) = 10\%$, $P(B) = 20\%$, $P(C) = 50\%$
e. $P(A) = 12.5\%$, $P(B) = 25\%$, $P(C) = 62.5\%$

(1 mark)

62. Experiment 1 results in the mutually exclusive and collectively exhaustive events **A**, **B** and **C**. Experiment 2 results in the mutually exclusive and collectively exhaustive events **D** and **E**. The events resulting from experiment 1 are independent of the events resulting from experiment 2 and vice versa. The joint probabilities of the events that may result from these two experiments are given below: [< Answer >](#)

	A	B	C
D	0.30	0.18	?
E	0.20	0.12	0.08

What is the joint probability of the events **C** and **D** occurring?

- a. 12% b. 20% c. 32% d. 40% e. 60%

(1 mark)

63. Two fair dice are thrown. What is the probability that one of them gives an even number less than 5, and the other one gives an odd number less than 4? [< Answer >](#)

- a. $\frac{1}{9}$ b. $\frac{2}{9}$ c. $\frac{1}{3}$ d. $\frac{4}{9}$ e. $\frac{5}{9}$

(1 mark)

64. A box contains one green ball and three blue balls. A second box contains two green balls and four blue balls. A third box contains three green balls and one blue ball. One of the three boxes is selected at random and a ball is randomly taken out of it. [< Answer >](#)

What is the likelihood that the ball is green?

- a. $\frac{5}{12}$ b. $\frac{1}{9}$ c. $\frac{4}{9}$ d. $\frac{1}{4}$ e. $\frac{3}{4}$

(1 mark)

[< Answer >](#)

65. In a certain part of a city, the number of residents is 6,000. Out of the 6,000 residents, 1,440 are above 30 years of age, and 3,600 are females. Out of the 1,440 residents who are above 30 years of age, 240 are females.

What is the probability that a resident selected at random is either a female or over 30 years of age?

- a. 0.04 b. 0.24 c. 0.60 d. 0.80 e. 0.90

(1 mark)

[< Answer >](#)

66. **A** and **B** are mutually exclusive and collectively exhaustive events. Both **A** and **B** are dependent on event **C**.

$$P(\mathbf{A} \text{ and } \mathbf{C}) = 0.48, \quad P(\mathbf{B} \text{ and } \mathbf{C}) = 0.32$$

What is the probability of event **B** happening if event **C** happens?

- a. 0.32 b. 0.40 c. 0.48 d. 0.60 e. 0.80

(1 mark)

[< Answer >](#)

67. The sales of a company during the year 2000 was Rs.74 lakhs and during the year 2002 was Rs.90 lakhs. The sales of the company during the year 2001 is not known.

What is the estimated value of sales for the year 2001, using the interpolation method?

- a. Rs.78 lakhs b. Rs.82 lakhs c. Rs.84 lakhs d. Rs.86 lakhs e. Rs.88 lakhs

(1 mark)

[< Answer >](#)

68. What is the value of **K** in the following series?

$$2 + 8 + 14 + 20 + 26 + \dots + K = 574$$

a. 70 b. 74 c. 80 d. 86 e. 90

(2 marks)

[< Answer >](#)

69. From twenty tickets marked with the first twenty numerals, one ticket is drawn at random. What is the probability that the numeral marked on it is a multiple of 3 or of 5?

- a. $\frac{1}{20}$ b. $\frac{1}{10}$ c. $\frac{3}{10}$ d. $\frac{9}{20}$ e. $\frac{1}{2}$

(2 marks)

[< Answer >](#)

70. The ratio of the likelihood that an individual **A**, who is 35 years old now, will live up to an age of 65, to the likelihood that he will die before 65 is 7:9. The ratio of the likelihood that another individual **B**, who is 45 years old now, will live up to an age of 75, to the likelihood that he will die before 75 is 2:3.

What is the likelihood that at least one of these individuals will be alive 30 years hence?

- a. $\frac{9}{16}$ b. $\frac{3}{5}$ c. $\frac{27}{80}$ d. $\frac{35}{80}$ e. $\frac{53}{80}$

(3 marks)

[< Answer >](#)

71. Experiment 1 results in the mutually exclusive and collectively exhaustive events **A**, **B** and **C**. Experiment 2 results in the mutually exclusive and collectively exhaustive events **D** and **E**. The joint probabilities of the events that may result from these two experiments are given below:

	A	B	C
D	0.12	0.075	0.12
E	0.28	0.225	0.18

Which of the following statements are **correct**?

- | | |
|----------------------|-----------------------|
| I. $P(A) = 0.30$ | II. $P(B) = 0.40$ |
| III. $P(C) = 0.30$ | IV. $P(D) = 0.685$ |
| V. $P(E) = 0.685$ | VI. $P(D C) = 0.25$ |
| VII. $P(E B) = 0.75$ | VIII. $P(D A) = 0.30$ |
- IX. **D** is independent of the events **A**, **B** and **C** respectively.
X. **E** is dependent on the events **A**, **B** and **C** respectively.

- a. (I), (IV) and (VI)
b. (IV), (VI), (VII) and (IX)
c. (II), (III), (IV), (VI) and (IX)
d. (III), (V), (VII), (VIII) and (X)
e. (I), (II), (IV), (VI) and (IX)

(3 marks)

[< Answer >](#)

72. ${}^nC_{10} \cdot 8! = {}^nP_8$

${}^nC_3 =$

- a. 153 b. 816 c. 252 d. 306 e. 4896

(2 marks)

[< Answer >](#)

73. A box contains 20 light bulbs, 5 of which are known to be defective. Three light bulbs are selected at random without replacement. Which of the following is/are **correct**?

- a. Probability that all the three bulbs selected at random are defective = $\frac{1}{1140}$
b. Probability that exactly one of the three bulbs selected at random is defective = $\frac{7}{76}$
c. Probability that at least one of the three bulbs selected at random is defective = $\frac{137}{228}$
d. Probability that at least two of the three bulbs selected at random are not defective = $\frac{91}{228}$
e. Probability that at least one of the three bulbs selected at random is not defective = $\frac{113}{114}$

(3 marks)

[< Answer >](#)

74. **P** and **Q** are the arithmetic mean and geometric mean respectively between the numbers **M** and **N**.

Which of the following is/are **correct**?

- | | |
|---|--|
| I. $M = \left[P^2 - Q^2 \right]^{\frac{1}{2}}$ | II. $N = \left[P^2 - Q^2 \right]^{\frac{1}{2}}$ |
| III. $M = P - \left[P^2 - Q^2 \right]^{\frac{1}{2}}$ | IV. $N = P + \left[P^2 - Q^2 \right]^{\frac{1}{2}}$ |
| V. $M = P + \left[P^2 - Q^2 \right]^{\frac{1}{2}}$ | VI. $N = P - \left[P^2 - Q^2 \right]^{\frac{1}{2}}$ |

$$\text{VII. } M = Q + \left[P^2 - Q^2 \right]^{\frac{1}{2}}$$

$$\text{VIII. } N = Q - \left[P^2 - Q^2 \right]^{\frac{1}{2}}$$

- a. I only b. II only c. III and IV d. V and VI e. VII and VIII
- (3 marks)**

75. **S** is the sum and **P** is the product of the first **n** terms in a Geometrical Progression (**G.P.**). **R** is the sum of the reciprocals of the first **n** terms of the **G.P.** **Sⁿ** =

- a. P^2 b. R^n c. P^n d. $P \cdot R^n$ e. $P^2 R^n$

(2 marks)

76. There are two groups of boxes. The first group contains three boxes each containing five yellow balls and two red balls. The second group contains two boxes each containing one yellow ball and four red balls. A box is selected at random out of the two groups of boxes and a ball is picked at random. If the ball that is picked is a red ball then, what is the probability that the ball is picked from the first group of boxes?

- a. $\frac{2}{21}$ b. $\frac{4}{5}$ c. $\frac{8}{25}$ d. $\frac{86}{175}$ e. $\frac{15}{43}$

(3 marks)

77. The formulation of a linear programming problem is given below:

$$\begin{aligned} \text{Maximize : } & Z = 25X + 35Y \\ \text{Subject to : } & 4X + 8Y \leq 400 \\ & 4X + 4Y \leq 280 \\ & 6X + 3Y \leq 360 \\ & X \geq 0 \\ & Y \geq 0 \end{aligned}$$

Solve the problem graphically and state which of the following is/are **correct**:

- I. The feasible region is bounded by the points $(X = 30, Y = 40)$, $(X = 20, Y = 50)$, $(X = 50, Y = 0)$, $(X = 0, Y = 60)$ and $(X = 0, Y = 0)$.
- II. The feasible region is bounded by the points $(X = 20, Y = 50)$, $(X = 0, Y = 50)$, $(X = 0, Y = 60)$ and $(X = 0, Y = 0)$.
- III. The feasible region is bounded by the points $(X = 30, Y = 50)$, $(X = 40, Y = 60)$, $(X = 50, Y = 60)$, $(X = 30, Y = 60)$ and $(X = 0, Y = 0)$.
- IV. The feasible region is bounded by the points $(X = 60, Y = 20)$, $(X = 0, Y = 20)$, $(X = 50, Y = 30)$ and $(X = 30, Y = 40)$.
- V. The feasible region is bounded by the points $(X = 60, Y = 0)$, $(X = 50, Y = 20)$, $(X = 40, Y = 30)$, $(X = 0, Y = 50)$ and $(X = 0, Y = 0)$.
- VI. The optimal solution to the problem is $(X = 0, Y = 0)$.
- VII. The optimal solution to the problem is $(X = 60, Y = 0)$.
- VIII. The optimal solution to the problem is $(X = 50, Y = 20)$.
- IX. The optimal solution to the problem is $(X = 0, Y = 50)$.
- X. The optimal solution to the problem is $(X = 40, Y = 30)$.

- a. I and VI b. II and VII c. III and VIII d. IV and IX e. V and X

(2 marks)

78. ${}^{n+1}C_r + {}^{n+1}C_{r-1} =$

[< Answer >](#)

[< Answer >](#)

[< Answer >](#)

[< Answer >](#)

$$a. {}^{n+1}P_r \quad b. {}^{n+1}P_{r-1} \quad c. \frac{{}^{n+2}P_r}{(r-1)!} \quad d. \frac{{}^{n+2}P_r}{r!} \quad e. \frac{{}^{n+2}P_r}{(r+1)!}$$

(2 marks)

[< Answer >](#)

79. Alpha industries Ltd. has 120 engineers on its rolls. Out of these 120 engineers, some are graduates and the remaining are post-graduates. The following details pertaining to the age-group and qualifications of the engineers have been provided by the personnel department of the company:

Age group (years)	Qualification		Total
	Graduate level	Post-graduate level	
Below 30	54	6	60
30 to 40	12	18	30
Above 40	24	6	30
Total	90	30	120

One engineer is selected at random.

Which of the following is **correct**?

- The probability that the engineer has a graduate level qualification is 0.60
- The probability that the engineer has a post-graduate level qualification given that his age is over 40 years, 0.10
- The probability that the engineer's age is below 30 years, given that he has a graduate level qualification, is 0.75
- The probability that the age of the engineer is between 30 and 40 years, and he has a graduate level qualification, is 0.20
- The probability that the age of the engineer is between 30 and 40 years given that he has a post-graduate level qualification, is 0.60.

(3 marks)

END OF SECTION B

Suggested Answers

Quantitative Methods – I (131) : January 2004

1. Answer :

Reason : If all the terms of an arithmetic progression are multiplied by a constant the resulting terms will always form an arithmetic progression with the first term multiplied by the constant as well as the common difference multiplied by the constant. The resulting series will neither be in a geometric series or a harmonic series because nature of the resulting terms will not satisfy their requirements.

[< TOP >](#)

2. Answer :

- Reason : a. The consecutive terms of the G.P. will be in increasing order if the first term in a geometric progression is greater than one and the common ratio is more than 1.
- b. The consecutive terms of the G.P. will be in decreasing order if the first term in a geometric progression is greater than one and the common ratio is less than 1.
- c. If the first term in a geometric progression is greater than one and the common ratio is less than 1, then the consecutive terms will not be the

[< TOP >](#)

same.

- d. All the consecutive terms will be less than 1 if the first term as well as the common ratio is less than 1.
- e. All the consecutive terms will be greater than 1 if the first term as well as the common ratio is more than 1. However a decreasing G.P. may still have all the terms greater than 1; this depends upon the number of terms in the G.P.

3. Answer :

[< TOP](#)
[>](#)

Reason : The reciprocal of the terms in a harmonic progression are in arithmetic progression. And vice versa. Therefore the reciprocals of a H.P. cannot be in H.P. There is no such connection between the H.P. and the G.P. If the terms in the corresponding A.P. are in the increasing order then the terms of the H.P. will be in the decreasing order and vice versa. Hence there is no reason why terms of a H.P. will always be in increasing or decreasing order.

4. Answer :

[< TOP](#)
[>](#)

Reason : a. The set of whole numbers includes the set of natural numbers.
b. The set of whole numbers does not include the set of rational numbers.
c. The set of whole numbers does not include the set of negative numbers.
d. The set of whole numbers does not include the set of complex numbers.
e. The set of whole numbers does not include the set of irrational numbers.

5. Answer :

[< TOP](#)
[>](#)

Reason : a. This is not a property of the real numbers
b. According to the inverse property of addition for every real number there exists another number such that the sum of the two real numbers is equal to 0.
c. This is the identity property of addition.
d. This is the inverse property of multiplication.
e. This is the identity property of multiplication.

6. Answer :

[< TOP](#)
[>](#)

Reason : a. This interval is bounded on the lower side but unbounded on the upper side.
b. This interval is bounded on the upper side but unbounded on the lower side.
c. This interval is bounded on the lower side but unbounded on the upper side.
d. This interval is bounded on the upper side but unbounded on the lower side.
e. This interval is bounded on the upper as well as lower sides.

7. Answer :

[< TOP](#)
[>](#)

Reason : If $X = p^m$ and $Y = p^n$, then the product of X and Y is equal to $c. p^{m+n}$

8. Answer : [< TOP](#)
[>](#)
Reason : The logarithm of a number is equal to the exponent to which a given base must be raised in order to get the number. The logarithm of a number is not expressed with respect to base zero or one. The logarithm of a number may be expressed with respect to base 10 or other positive real number not equal to 1.
9. Answer : [< TOP](#)
[>](#)
Reason : A function is a rule or correspondence which always associates to each number x in a set A , a unique number $f(x)$ in a set B .
10. Answer : [< TOP](#)
[>](#)
Reason : In a two dimensional plane the graph of a function of the form $y = a + bx$ is a straight line.
11. Answer : [< TOP](#)
[>](#)
Reason : The accumulated amount at the end of t years, of a given amount of investment, under compound interest, is an example of exponential function.
12. Answer : [< TOP](#)
[>](#)
Reason : For any function $f(x)$ the limit of $f(x)$ as x approaches a value a is a value L , such that $f(x)$ approaches L as x approaches a .
13. Answer : [< TOP](#)
[>](#)
Reason : a. The derivative of a function $f(x)$ at $x = a$ may not be equal to $f(a)$.
b. The derivative of a function $f(x)$ at $x = a$ may not be equal to a .
c. The derivative of $f(x)$ at $x = a$ is the rate of change in the value of $f(x)$ at $x = a$.
d. The derivative of a function $f(x)$ at $x = a$ is not the limit of $f(x)$ as x approaches a .
e. The derivative of a function $f(x)$ at $x = a$ is not the ratio of $f(a)$ to a .
14. Answer : [< TOP](#)
[>](#)
Reason : If the derivative of a function $f(x)$ is zero, then it implies that $f(x)$ has a constant value for all values of x .
15. Answer : [< TOP](#)
[>](#)
Reason : For a function **$f(x)$** the first derivative is positive and the second derivative is negative at **$x = a$** . This means that $f(x)$ is increasing at a decreasing rate at **$x = a$** . For a function **$f(x)$** to be minimum or maximum the first derivative should be equal to zero.
16. Answer : [< TOP](#)
[>](#)
Reason : a. If the first derivative of **$f(x)$** is a constant for all values of x then it indicates that the function changes at a constant rate.
b. A function **$f(x)$** is said to be monotonically decreasing if the first derivative of $f(x)$ is negative for all values of x .
c. A function **$f(x)$** is said to be monotonically increasing if the first

derivative of $f(x)$ is

positive for all values of x .

d. If the first derivative of $f(x)$ is zero for all values of x then it indicates that the function has a constant value.

e. This is not the condition for a monotonically increasing function.

17. Answer :

[< TOP](#)
[>](#)

Reason : If the first derivative of a function $f(x)$ is zero at $x = a$ then $f(x)$ will be maximum at $x = a$ if the second derivative is negative at $x = a$. If the first derivative of a function $f(x)$ is zero at $x = a$ then $f(x)$ will be minimum at $x = a$ if the second derivative is positive at $x = a$.

In the given condition it cannot be inferred that $f(x)$ is zero at $x = a$.

18. Answer :

[< TOP](#)
[>](#)

Reason : a. In a logarithmic function the logarithmic operation is applied on the independent variable.

b. In a logarithmic function the base of the logarithm may be any number other than 10.

c. In a logarithmic function the independent variable cannot be negative.

d. In a logarithmic function the base of the logarithm cannot be zero or negative numbers.

19. Answer :

[< TOP](#)
[>](#)

Reason : Three simultaneous equations involving three variables can be solved if the three equations can be satisfied when the three variables respectively assume the same values in each of the three equations.

20. Answer :

[< TOP](#)
[>](#)

Reason : a. The technique of linear programming may be applied for decision-making when there is an objective to be achieved

b. The technique of linear programming may be applied for decision-making when the objective can be expressed in the form of a linear function.

c. The technique of linear programming may be applied for decision-making when there are alternative courses of action.

d. The technique of linear programming need not be applied for decision-making when there is no constraint in the supply of resources.

e. The technique of linear programming may be applied for decision-making when there are multiple decision variables to which resources must be allocated.

21. Answer :

[< TOP](#)
[>](#)

Reason : The following are true for any LPP:

a. The contribution of each unit of the decision variables towards to the objective to be achieved is known.

- b. The consumption of resources by each unit of the decision variables is known.
- c. The values of the decision variables in the optimal solution may be fractional numbers.
- d. Linear programming is used either to maximize or to minimize the value of the objective function.
- e. The optimal solution to any linear programming problem is one of the possible feasible solutions.

22. Answer :

[< TOP](#)
[>](#)

Reason :a. The objective function specifies the objective to be achieved by solving the LPP.

- b. The limitations on the use of resources are represented by the structural constraints.
- c. Nonnegativity constraints are used to prevent the decision variables from assuming negative values.
- d. Decision variables are the variables, which contribute to the value of the objective function, and their values are determined by solving the LPP.
- e. Solutions represent the specific combinations of values of the decision variables.

23. Answer :

[< TOP](#)
[>](#)

Reason :The feasible region is the set off all points which satisfy all the constraints in the LPP.

24. Answer :

[< TOP](#)
[>](#)

Reason :The following are true with regard to the graphical method of solving LPPs:

- a. It is applicable when there are two decision variables.
- b. The decision variables are represented by the horizontal and vertical axes.
- c. Straight lines are used to demarcate the feasible region.
- d. The feasible region shows the solutions that satisfy all the constraints.
- e. The corner points of the feasible region may not include the origin.

25. Answer :

[< TOP](#)
[>](#)

Reason :In the graphical method of solving linear programming problems if there is a unique optimal solution, then the optimal solution is located at one of the corner points of the feasible region.

26. Answer :

[< TOP](#)
[>](#)

Reason :The following are **true** with regard to simplex method of solving linear programming problems:

- a. This method can be applied for solving problems involving two decision variables.
- b. At the optimal solution all the decision variables assume non-negative values.
- c. Slack variables are used convert 'less than or equal to' type constraints into equations.
- d. At the optimal solution all the slack variables assume non-negative values.
- e. The $Z_j - C_j$ value at the bottom of the solution column shows the value of the objective function for any solution.

27. Answer :

[< TOP](#)
[≥](#)

Reason :The following are true with regard to the simplex method of solving linear programming problems:

- a. It involves an iterative procedure for arriving at the optimal solution.
- b. Slack variables are used to represent the unused resources.
- c. Slack variables make zero contribution towards the objective to be achieved.
- d. The $Z_j - C_j$ values indicate the variable to enter solution.
- e. The $Z_j - C_j$ values indicate whether the solution is optimal or not.

28. Answer :

[< TOP](#)
[≥](#)

Reason :The following are true with regard to histograms:

- a. The classes are represented by the base of the rectangles.
- b. The frequencies are represented by the heights of the rectangles.
- c. If the classes are of equal width then the bases of the rectangles will be of equal length.
- d. The tallest rectangle in a histogram represents the class with the highest frequency.

29. Answer :

[< TOP](#)
[≥](#)

Reason :a. The skewness of a distribution indicates the absence of symmetry in the distribution.

- b. The consistency of the data in the distribution is indicated by the coefficient of variation.
- c. The dispersion of the data in the distribution is indicated by the standard deviation.
- d. The central tendency of the distribution is indicated by the mean, median or mode.

30. Answer :

[< TOP](#)
[≥](#)

Reason :In a positively skewed distribution the majority of the observations are

concentrated towards the lower end of the scale. In a negatively skewed distribution majority of the observations are concentrated towards the higher end of the scale. In a skewed distribution the distribution of the data is not symmetrical.

31. Answer :

[< TOP](#)
[>](#)

Reason :The standard deviation represents the scatter of the values in a data set.

Arithmetic mean, geometric mean, harmonic mean and median are measures of central tendency.

32. Answer :

[< TOP](#)
[>](#)

Reason :Range (= Highest value – Lowest value) is based only on two observations in a data set.

Arithmetic mean, harmonic mean, mean absolute deviation and standard deviation are based on all the observations.

33. Answer :

[< TOP](#)
[>](#)

Reason :a. Range is based on the two extreme values in the data set. Hence it is affected by the extreme values in the data set.

b. Arithmetic mean is based on all the values. Hence this is also affected by the presence of extreme of values.

c. Standard deviation is based on all the values. Hence this is also affected by the presence of extreme of values.

d. Variance is based on all the values. Hence this is also affected by the presence of extreme of values.

e. Median is middle value of a data array. So it is not affected by the presence of the extreme values.

34. Answer :

[< TOP](#)
[>](#)

Reason :Coefficient of variation is a relative measure of dispersion. Range, quartile deviation, standard deviation and mode are absolute measures of dispersion. When every observation in the data set is divided by a constant both the standard deviation and the mean of the resulting data set will be divided by the constant. Since coefficient of variation is the ratio standard deviation to the mean, the coefficient of variation of the resulting data set will be same as the original data set. Since the other measures are absolute measures, they will permanently change due to the modification.

35. Answer :

[< TOP](#)
[>](#)

Reason :If a constant is subtracted from every observation in a data set then the range of the resulting set of values will be equal to the range of the original data set.

36. Answer :

[< TOP](#)
[>](#)

Reason :If every observation in a data set is increased by a constant then the coefficient of variation of the resulting set of values will be less than the coefficient of variation of the original data set because the numerator

remains the same whereas the denominator increases.

37. Answer :

[< TOP](#)
[>](#)

Reason :If two events A and B are such that their joint probability is equal to the product of their marginal probabilities then A and B are independent events.

38. Answer :

[< TOP](#)
[>](#)

Reason :If two events A and B are independent then, the conditional probability of event A given event B is equal to marginal probability of event A because the occurrence of event B does not influence the occurrence of event A.

39. Answer :

[< TOP](#)
[>](#)

Reason :The events for which the Baye's theorem may be applied for computing posterior probabilities must be mutually exclusive and collectively exhaustive.

40. Answer :

[< TOP](#)
[>](#)

Reason :If certain events are mutually exclusive and collectively exhaustive then only one of them can happen at a time and the entire sample space is divided among the two events. Hence the sum of their probabilities will be equal to 1.00.

41 Answer :

[< TOP](#)
[>](#)

Reason :Range = Highest value – Lowest value = 20 – 7 = 13

42 Answer :

[< TOP](#)
[>](#)

Reason :Arithmetic mean = $\frac{\sum x}{n} = \frac{3+8+5+6+10+7}{6} = 6.5$

43 Answer :

[< TOP](#)
[>](#)

Reason :Mode is the observation which has the highest frequency.

∴ Mode of the given data set is 7, because 7 has the highest frequency (4).

44 Answer :

[< TOP](#)
[>](#)

Reason :Coefficient of variation = $\frac{\text{Standard deviation}}{\text{Mean}} \times 100$

$$= \frac{\sqrt{\text{Variance}}}{\text{Mean}} \times 100$$

$$\text{or } 20 = \frac{\sqrt{16}}{\text{Mean}} \times 100$$

$$\text{or Mean} = \frac{4}{20} \times 100 = 20$$

45 Answer :

[< TOP](#)
[>](#)

$$\text{Reason : Variance} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

If every item is multiplied by 2 then mean of the resulting data set will be $2\bar{x}$ and the variance of the resulting data set

$$\begin{aligned} &= \frac{1}{n-1} \sum (2x - 2\bar{x})^2 = \frac{1}{n-1} \sum 2^2 (x - \bar{x})^2 = 4 \times \frac{1}{n-1} \sum (x - \bar{x})^2 \\ &= 4 \times \text{Variance of the original data set} \\ &= 4 \times 9 = 36. \end{aligned}$$

46 Answer :

$$\text{Reason : Given : } \sum x = 120 \quad s = 3 \quad n = 10$$

$$\therefore \bar{x} = \frac{\sum x}{n} = \frac{120}{10} = 12$$

After 3 added to every observation, mean of the resulting data set = $\bar{x} + 3 = 12 + 3 = 15$

Standard deviation of the resulting data set will be unchanged, i.e. = 3.

$$\therefore \text{Coefficient of variation of the resulting data set} = \frac{3}{15} \times 100 = 20\%$$

47 Answer :

$$\text{Reason : } S^2 = \frac{1}{n-1} \left[\sum x^2 - n\bar{X}^2 \right] = \frac{1}{n-1} \left[\sum x^2 - \frac{(\sum x)^2}{n} \right]$$

$$\sum_{i=1}^{10} x_i = \sum_{i=1}^4 x_i + \sum_{i=5}^{10} x_i$$

$$\sum_{i=1}^4 x_i = 16 \times 4 = 64$$

$$\sum_{i=5}^{10} x_i = 11 \times 6 = 66$$

$$\therefore \sum_{i=1}^{10} x_i = 64 + 66 = 130$$

$$\sum_{i=1}^{10} x_i^2 = 1726 \quad (\text{given})$$

$$\therefore n = 10$$

$$\therefore S^2 = \frac{1}{9} \left[1726 - \frac{(130)^2}{10} \right] = 4$$

$$\therefore S = \sqrt{4} = 2$$

[TOP](#)

[TOP](#)

48 Answer :

[TOP](#)

Reason : Average time taken now = $\frac{(50 \times 4) + (125 \times 5) + (75 \times 6)}{(50 + 125 + 75)} = 5.1$ hours

Average time taken after three years = $\frac{4(50 + 125) + (75 \times 5)}{(50 + 125) + 75} = 4.3$ hours

Change in time taken = $5.1 - 4.3 = 0.80$ hours reduction.

49 Answer :

[TOP](#)

Reason : Let the following notations be used:

N_1 : Number of sales men with 5 years experience

N_2 : Number of sales men with 10 years experience

$\therefore \bar{X}_2$: Average salary of sales men with 10 years experience.

By the question –

$$\frac{N_1 \times 7000 + N_2 \bar{X}_2}{N_1 + N_2} = 9000$$

$$\text{or } \frac{N_1}{N_1 + N_2} (7000) + \frac{N_2}{N_1 + N_2} \bar{X}_2 = 9000$$

$$\text{or } (0.6 \times 7000) + (1 - 0.6) \bar{X}_2 = 9000$$

$$\text{or } 0.4 \bar{X}_2 = 9000 - 4200 = 4800$$

$$\text{or } \bar{X}_2 = \frac{4800}{0.4} = \text{Rs. } 12000.$$

50 Answer :

[TOP](#)

Reason : Median marks = $\left[\frac{(N+1)/2 - (F+1)}{f_m} \right] W + L_m$

Position of median = $\frac{60+1}{2} = 30.5\text{th value}$

Class	Frequency	Cumulative frequency
0-20	6	6
20-40	14	20
40-60	18	38
60-80	12	50
80-100	8	58

100-120	2	60
	60	

Median class is 40 – 60

$$N = 60$$

$$L_M = 40$$

$$F = 20$$

$$f_m = 18$$

$$W = 20$$

$$\therefore \text{Median} = \left[\frac{61/2 - (20+1)}{18} \right] 20 + 40 = 50.6$$

51 Answer :

$$\text{Reason : } \frac{y}{3} + \frac{2}{5}x = 2$$

$$\Rightarrow 5y + 6x = 30$$

$$\Rightarrow y = \frac{30}{5} - \frac{6x}{5}$$

$$\text{or } y = 6 - \frac{6}{5}x$$

$$\therefore \text{Intercept on Y axis} = 6$$

$$\text{Slope} = \frac{-6}{5}$$

52 Answer :

Reason : The given series is in G.P with $a = 3$ and $r = 2$

$$\therefore t_8 = ar^{n-1} = 3 \times (2)^{8-1} = 3 \times 2^7 = 384.$$

53 Answer :

$$\text{Reason : } \log_2 8 + \log_3 27 = 3 + 3 = 6$$

54 Answer :

$$\begin{aligned} \text{Reason : Sum of an A.P with 'n' terms, } S_n &= \frac{n}{2}(a + t_n) \\ &= \frac{8}{2}(6 + 34) = 160 \end{aligned}$$

55 Answer :

$$\text{Reason : } \lim_{t \rightarrow 0} \frac{2 - \sqrt{4 - t^2}}{t^2} = \lim_{t \rightarrow 0} \frac{(2 + \sqrt{4 - t^2})(2 - \sqrt{4 - t^2})}{t^2(2 + \sqrt{4 - t^2})}$$

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$$\begin{aligned}
&= \lim_{t \rightarrow 0} \frac{4 - (4 - t^2)}{t^2 \left(2 + \sqrt{4 - t^2} \right)} \\
&= \lim_{t \rightarrow 0} \frac{t^2}{t^2 \left(2 + \sqrt{4 - t^2} \right)} \\
&= \lim_{t \rightarrow 0} \frac{1}{2 + \sqrt{4 - t^2}} \\
&= \frac{1}{2 + 2} = \frac{1}{4}
\end{aligned}$$

56 Answer :

[TOP](#)

Reason : $u = \ln \left(t + \sqrt{t^2 + 4} \right)$

$$\begin{aligned}
\frac{du}{dt} &= \frac{1}{t + \sqrt{t^2 + 4}} \times \frac{d}{dt} \left(t + \sqrt{t^2 + 4} \right) = \frac{1}{t + \sqrt{t^2 + 4}} \times \left\{ 1 + \frac{1}{2} (t^2 + 4)^{\frac{1}{2} - 1} \times (2t + 0) \right\} \\
&= \frac{1}{t + \sqrt{t^2 + 4}} \times \left\{ 1 + \frac{t}{\sqrt{t^2 + 4}} \right\} \\
&= \frac{t + \sqrt{t^2 + 4}}{\sqrt{t^2 + 4}} \times \frac{1}{t + \sqrt{t^2 + 4}} = \frac{1}{\sqrt{t^2 + 4}} = (t^2 + 4)^{-\frac{1}{2}}
\end{aligned}$$

57 Answer :

[TOP](#)

Reason : $\lim_{x \rightarrow \infty} \frac{x^2 - 3x + 2}{3x^2 - 4x}$

$$\begin{aligned}
&= \lim_{x \rightarrow \infty} \frac{\frac{1}{x^2} (x^2 - 3x + 2)}{\frac{1}{x^2} (3x^2 - 4x)} \\
&= \lim_{x \rightarrow \infty} \frac{1 - \frac{3}{x} + \frac{2}{x^2}}{3 - \frac{4}{x}} = \frac{1}{3}
\end{aligned}$$

Because as $x \rightarrow \infty$, $\frac{3}{x}$ and $\frac{2}{x^2}$ in the numerator tend to zero, and $\frac{4}{x}$ in the denominator tends to zero.

58 Answer :

Reason : $u = 2t^3 - 21t^2 + 36t + 50$

$$\frac{du}{dt} = 2(3t^2) - 21(2t) + 36 = 6t^2 - 42t + 36$$

The common condition for both maxima and minima is $\frac{du}{dt} = 0$

$$\therefore 6t^2 - 42t + 36 = 0$$

$$\text{or } t^2 - 7t + 6 = 0$$

$$\text{or } t^2 - (6+1)t + 6 = 0$$

$$\text{or } t^2 - 6t - t + 6 = 0$$

$$\text{or } t(t-6) - (t-6) = 0$$

$$\text{or } (t-1)(t-6) = 0$$

$$\Rightarrow t = 1 \text{ or } t = 6$$

$$\text{For maxima } \frac{d^2u}{dt^2} < 0$$

$$\text{For minima } \frac{d^2u}{dt^2} > 0$$

$$\frac{d^2u}{dt^2} = 12t - 42$$

$$\text{At } t = 1 \quad \frac{d^2u}{dt^2} = 12 \times 1 - 42 = -30 < 0 \therefore u \text{ is maximum at } t = 1$$

$$\text{At } t = 6 \quad \frac{d^2u}{dt^2} = 12 \times 6 - 42 = 30 > 0 \therefore u \text{ is minimum at } t = 6$$

59 Answer :

Reason : $\int_0^1 2x e^{x^2+5} dx = \left[e^{x^2+5} \right]_0^1 = \left[e^6 - e^5 \right] = e^5 (e - 1)$

$$\text{Since, } \frac{d}{dx} \left(e^{f(x)} \right) = f'(x) \cdot e^{f(x)}$$

$$\therefore \frac{d}{dx} \left(e^{x^2+5} \right) = (2x+0) e^{x^2+5} = 2x e^{x^2+5}$$

60 Answer :

Reason : If $f(x) = \ln g(x)$, then $f'(x) = \frac{g'(x)}{g(x)}$

[TOP](#)

[TOP](#)

[TOP](#)

$$\begin{aligned}\frac{d}{dx} \ln(x^3 + 4) &= \frac{3x^2}{x^3 + 4} \\ \therefore \int_1^2 \frac{3x^2}{x^3 + 4} dx &= \left[\ln(x^3 + 4) \right]_1^2 = \left[\ln(2^3 + 4) - \ln(1^3 + 4) \right] \\ &= [\ln 12 - \ln 5] = \ln\left(\frac{12}{5}\right)\end{aligned}$$

61 Answer :

Reason : $P(A) = x \quad \therefore P(B) = 2x \quad P(C) = 2.5 P(B) = 2.5(2x) = 5x$

Since A, B and C are mutually exclusive and collectively exhaustive,

$$P(A) + P(B) + P(C) = 1$$

$$\therefore x + 2x + 5x = 1$$

$$\text{or } 8x = 1$$

$$\text{or } x = \frac{1}{8} = 0.125 \quad \text{i.e. } 12.5\%$$

$$\begin{aligned}\therefore P(A) &= 12.5\% & P(B) &= 2 \times 12.5 = 25\% & P(C) &= 5 \times 12.5 \\ &= 62.5\%\end{aligned}$$

[TOP](#) [<](#) [>](#)

62 Answer :

Reason : $P(C \text{ and } D) = 1 - (0.30 + 0.20 + 0.18 + 0.12 + 0.08) = 0.12$ i.e. 12%.

[TOP](#) [<](#) [>](#)

63 Answer :

Reason : Probability that the first dice gives an even number less than 5 and the

$$\begin{aligned}\text{second dice gives an odd number less than 4} &= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}\end{aligned}$$

Probability that the second dice gives an even number less than 5 and the

$$\text{first dice gives an odd number less than 4} = \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$$

Since these events are mutually exclusive, the probability that one of the dice gives an even number less than 5 and the other one gives an odd

$$\text{number less than 4} = \frac{1}{9} + \frac{1}{9} = \frac{2}{9}$$

[TOP](#) [<](#) [>](#)

64 Answer :

Reason : The box is selected at random. So each box is equally likely to be selected.

$$\therefore \text{Probability of selecting a green ball from the first box} = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

[TOP](#) [<](#) [>](#)

Probability of selecting a green ball from the second box = $\frac{1}{3} \times \frac{2}{6} = \frac{1}{9}$

Probability of selecting a green ball from the third box = $\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

$\therefore$ The likelihood that the ball is green = $\frac{1}{12} + \frac{1}{9} + \frac{1}{4} = \frac{4}{9}$

65 Answer :

Reason : Let the following notation be used:

A : The resident is a female

B: The resident is above 30 years of age

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$= \frac{3600}{6000} + \frac{1440}{6000} - \frac{240}{6000} = 0.80$$

66 Answer :

Reason : $P(A \text{ and } C) = 0.48$

$P(B \text{ and } C) = 0.32$

$\therefore P(C) = P(A \text{ and } C) + P(B \text{ and } C) = 0.80$

$P(B \text{ and } C) = P(C) \cdot P(B/C)$

$$\text{Or, } P(B/C) = \frac{P(B \text{ and } C)}{P(C)} = \frac{0.32}{0.80} = 0.40$$

67 Answer :

Reason : Sales during the year 2001 = $74 + \frac{90 - 74}{(2002 - 2000)} \times (2001 - 2000)$
= Rs.82 lakhs.

68 Answer :

Reason : Let K be the nth term.

The given series is in A.P

First term, $a = 2$

Common difference, $d = 6$

$\therefore K = a + (n - 1)d = 2 + 6(n - 1)$

By the question:

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow 574 = \frac{n}{2} [2 \times 2 + (n-1)6]$$

$$\text{or } 574 = \frac{n}{2} [4 + 6(n-1)]$$

$$\text{or } 574 = \frac{n}{2} [4 + 6n - 6] = \frac{n}{2} [6n - 2]$$

$$\text{or } 1148 = 6n^2 - 2n$$

$$\text{or } 6n^2 - 2n - 1148 = 0$$

$$\text{or } 3n^2 - n - 574 = 0$$

$$\therefore n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 3 \times (-574)}}{2 \times 3}$$

$$= \frac{1 \pm \sqrt{1 + 6888}}{6}$$

$$= \frac{1+83}{6} = 14 \quad \text{or} \quad -\frac{82}{6}$$

Since the number of terms, n cannot be negative, $n = 14$

$$\therefore K = 2 + 6(14 - 1) = 80$$

69 Answer :

Reason : Probability that the numeral is a multiple of 3 (event A) = $\frac{6}{20}$

Probability that the numeral is a multiple of 5 (event B) = $\frac{4}{20}$

Probability that the numeral is a multiple of both 3 and 5 (event A and B)

$$= \frac{1}{20}$$

$\therefore$ Probability that the numeral is a multiple of 3 or 5,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{6+4-1}{20} = \frac{9}{20}.$$

70 Answer :

Reason : Let the following notations be used.

A: A lives upto 65 (= 35 + 30)

A': Complement of event A

B: B lives upto 75 (= 45 + 30)

B': Complement of event B

$$P(A) + P(A') = 1$$

$$\frac{P(A)}{P(A')} = \frac{7}{9} \quad \text{or} \quad \frac{P(A')}{P(A)} = \frac{9}{7} \quad \text{or} \quad P(A') = \frac{9}{7} P(A)$$

$$\therefore P(A) + \frac{9}{7} P(A) = 1$$

[TOP](#)

[TOP](#)

$$\text{or } 7P(A) + 9P(A) = 7 \quad \text{or} \quad 16P(A) = 7 \quad \text{or} \quad P(A) = \frac{7}{16}$$

Similarly :

$$P(B) + P(B') = 1$$

$$\frac{P(B)}{P(B')} = \frac{2}{3} \quad \text{or} \quad \frac{P(B')}{P(B)} = \frac{3}{2} \quad \text{or} \quad P(B') = \frac{3}{2}P(B)$$

$$\therefore P(B) + \frac{3}{2}P(B) = 1$$

$$\text{or } 2P(B) + 3P(B) = 2 \quad \text{or } 5P(B) = 2 \quad \text{or } P(B) = \frac{2}{5}$$

$\therefore$ Probability that at least one of them will be alive 30 years hence

$$= P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$= \frac{7}{16} + \frac{2}{5} - \left(\frac{7}{16} \times \frac{2}{5} \right) = \frac{53}{80}$$

71 Answer :

$$\text{Reason : } P(A) = P(A \text{ and } D) + P(A \text{ and } E) = 0.12 + 0.28 = 0.40$$

Similarly,

$$P(B) = 0.075 + 0.225 = 0.30$$

$$P(C) = 0.12 + 0.18 = 0.30$$

$$P(D) = P(A \text{ and } D) + P(B \text{ and } D) + P(C \text{ and } D) = 0.12 + 0.075 + 0.12 = 0.315$$

Similarly,

$$P(E) = 0.28 + 0.225 + 0.18 = 0.685$$

$$P(D|C) = \frac{P(C \text{ and } D)}{P(C)} = \frac{0.12}{0.30} = 0.40$$

$$P(E|B) = \frac{P(B \text{ and } E)}{P(B)} = \frac{0.225}{0.30} = 0.75$$

$$P(D|A) = \frac{P(A \text{ and } D)}{P(A)} = \frac{0.12}{0.40} = 0.30$$

$$P(D|B) = \frac{P(B \text{ and } D)}{P(B)} = \frac{0.075}{0.30} = 0.25$$

$$P(E|A) = \frac{P(A \text{ and } E)}{P(A)} = \frac{0.28}{0.40} = 0.70$$

[TOP](#)

$$P(E|C) = \frac{P(C \text{ and } E)}{P(C)} = \frac{0.18}{0.30} = 0.60.$$

From above we can see that:

$$P(D|A) \neq P(D) \quad P(D|B) \neq P(D) \quad P(D|C) \neq P(D)$$

$\therefore$ D is dependent on the events A, B and C respectively.

$$P(E|A) \neq P(E) \quad P(E|B) \neq P(E) \quad P(E|C) \neq P(E)$$

$\therefore$ E is dependent on the events A, B and C respectively.

72 Answer :

$$\text{Reason : } {}^nC_{10} \cdot 8! = {}^nP_8$$

$$\text{Or } {}^nC_{10} = \frac{{}^nP_8}{8!}$$

$$\text{Or, } {}^nC_{10} = {}^nC_8 \quad \left[\because {}^nC_r = \frac{{}^nP_r}{r!} \right]$$

$$\text{Or, } {}^nC_{n-10} = {}^nC_8 \quad \left[\because {}^nC_{n-r} = {}^nC_r \right]$$

$$\Rightarrow n - 10 = 8$$

$$\therefore n = 10 + 8 = 18$$

$$\therefore {}^nC_3 = {}^{18}C_3 = \frac{18!}{3!(18-3)!} = 816$$

[TOP](#)

73 Answer :

Reason : Total number of bulbs = 20

Number of defectives = 5

Number of non-defectives = 20 - 5 = 15

$$(a) \quad P(\text{all three bulbs are defective}) = \frac{{}^5C_3}{{}^{20}C_3} = \frac{10}{1140} = \frac{1}{114}$$

$$(b) \quad P(\text{exactly one of the three bulbs is defective}) = \frac{{}^5C_1 \times {}^{15}C_2}{{}^{20}C_3} = \frac{5 \times 105}{1140} = \frac{35}{76}$$

$$(c) \quad P(\text{at least one of the three bulbs is defective}) = 1 - P(\text{none of the bulbs is defective})$$

$$= 1 - \frac{{}^{15}C_3}{{}^{20}C_3} = 1 - \frac{455}{1140} = \frac{137}{228}$$

$$(d) \quad P(\text{at least two of the three bulbs are not defective}) = P(2 \text{ are not defective}) + P(3 \text{ are not defective})$$

[TOP](#)

$$= \frac{{}^{15}C_2 \times {}^5C_1}{{}^{20}C_3} + \frac{{}^{15}C_3}{{}^{20}C_3} = \frac{5 \times 105}{1140} + \frac{455}{1140} = \frac{49}{57}$$

(e) P(at least one of the three bulbs is not defective)

= 1 - P (All the three bulbs are defective)

$$= 1 - \frac{{}^5C_3}{{}^{20}C_3} = 1 - \frac{10}{1140} = \frac{113}{114}$$

74 Answer :

Reason : M, P and N are in A.P

$$\therefore P - M = N - P \Rightarrow 2P = M + N \Rightarrow P = \frac{M + N}{2} \dots\dots\dots(1)$$

$$\therefore P^2 = \frac{(M + N)^2}{4} = \frac{M^2 + 2MN + N^2}{4} \dots\dots\dots(2)$$

M, Q and N are in G.P

$$\therefore \frac{Q}{M} = \frac{N}{Q} \Rightarrow Q^2 = MN$$

$$\therefore P^2 - Q^2 = \frac{M^2 + 2MN + N^2}{4} - MN \dots\dots\dots(3)$$

$$= \frac{M^2 + 2MN + N^2 - 4MN}{4} = \frac{M^2 - 2MN + N^2}{4}$$

$$= \left[\frac{M - N}{2} \right]^2$$

$$\therefore \left[P^2 - Q^2 \right]^{\frac{1}{2}} = \frac{M - N}{2} \dots\dots\dots(4)$$

Adding equation (1) and (4):

$$P + \left[P^2 - Q^2 \right]^{\frac{1}{2}} = \frac{M + N + M - N}{2} = \frac{2M}{2} = M$$

$$\text{i.e. } M = P + \left[P^2 - Q^2 \right]^{\frac{1}{2}}$$

Subtracting equation (4) from equation (1):

$$P - \left[P^2 - Q^2 \right]^{\frac{1}{2}} = \frac{M + N - M + N}{2} = \frac{2N}{2} = N$$

$$\text{i.e. } N = P - \left[P^2 - Q^2 \right]^{\frac{1}{2}}$$

$\therefore$ V and VI are correct

75 Answer :

$$\text{Reason : } S = \frac{a(1-r^n)}{1-r}$$

$$P = a \times ar \times ar^2 \dots \times ar^{n-1} = a^n r^{1+2+\dots+n-1}$$

$$= a^n r^{\frac{n(n-1)}{2}}$$

$$[1+2+\dots+n-1] = \frac{(n-1)}{2} [2 \times 1 + (n-2)1] = \frac{n-1}{2} [2+n-2] = \frac{n(n-1)}{2}$$

$$\therefore P^2 = a^{2n} r^{n(n-1)}$$

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots + \frac{1}{ar^{n-1}}$$

$$= \frac{1}{a} \times \frac{\left[\left(\frac{1}{r} \right)^n - 1 \right]}{\frac{1}{r} - 1}$$

$$= \frac{r}{a} \times \frac{1-r^n}{(1-r)r^n}$$

$$\therefore \frac{S}{R} = \frac{a(1-r^n)}{(1-r)} \times \frac{a(1-r)r^n}{r(1-r^n)} = a^2 r^{n-1}$$

$$\therefore \frac{S^n}{R^n} = a^{2n} r^{n(n-1)} = P^2$$

$$\therefore S^n = P^2 R^n$$

76 Answer :

Reason : Group 1 contains 3 boxes and group 2 contains two boxes

$$\therefore P(G1) = \frac{3}{5}$$

$$P(G2) = \frac{2}{5}$$

Probability of selecting a red ball from group 1 = $P(R|G1) =$

$$\left(\frac{1}{3} \times \frac{2}{7} \right) + \left(\frac{1}{3} \times \frac{2}{7} \right) + \left(\frac{1}{3} \times \frac{2}{7} \right) = \frac{2}{7}$$

Probability of selecting a red ball from group 2 = $P(R|G2)$

$$\left(\frac{1}{2} \times \frac{4}{5} \right) + \left(\frac{1}{2} \times \frac{4}{5} \right) = \frac{4}{5}$$

$$\therefore P(G1 \text{ and } R) = P(G1).P(R|G1) = \frac{3}{5} \times \frac{2}{7}$$

$$= \frac{6}{35}$$

[TOP](#)

$$P(G2 \text{ and } R) = P(G2).P(R|G2) = \frac{2}{5} \times \frac{4}{5}$$

$$= \frac{8}{25}$$

$$\therefore P(R) = P(G1 \text{ and } R) + P(G2 \text{ and } R) = \frac{6}{35} + \frac{8}{25}$$

$$= \frac{86}{175}$$

$$\therefore P(G1|R) = \frac{P(G1 \text{ and } R)}{P(R)} = \frac{6/35}{86/175}$$

$$= \frac{15}{43}$$

77 Answer :

Reason : The LPP formulation can be rewritten as follows:

Maximize : $25X + 35Y$

Subject to : $X + 2Y \leq 100$

$X + Y \leq 70$

$2X + Y \leq 120$

$X \geq 0$

$Y \geq 0$

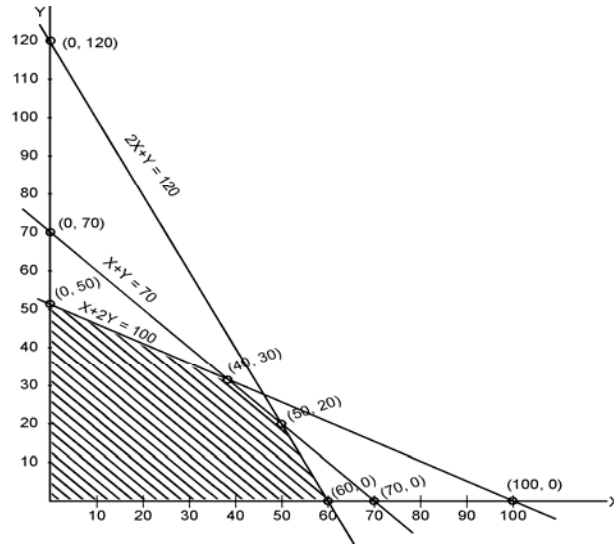
From the graph we can see that the feasible region is bounded by the points (0,0), (0,50), (40,30), (50,20) and (60,0)

Point	$Z = 25X + 35Y$
$X = 0, Y = 0$	$Z = 0$
$X = 0, Y = 50$	$Z = (25 \times 0) + (35 \times 50) = 1750$
$X = 40, Y = 30$	$Z = (25 \times 40) + (35 \times 30) = 2050$
$X = 50, Y = 20$	$Z = (25 \times 50) + (35 \times 20) = 1950$
$X = 60, Y = 0$	$Z = (25 \times 60) + (35 \times 0) = 1500$

The maximum value of the objective function is 2050 at the point (X = 40, Y = 30).

Hence the optimal solution to the LPP is (X = 40, Y = 30).

[TOP](#)



78 Answer :

Reason : ${}^{n+1}C_r + {}^{n+1}C_{r-1}$

$$= \frac{(n+1)!}{r!(n-r+1)!} + \frac{(n+1)!}{(r-1)!(n+1-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!} + \frac{(n+1)!}{(r-1)!(n-r+2)!}$$

$$= \frac{(n+1)!}{(r-1)!(n-r+1)!} \left[\frac{1}{r} + \frac{1}{n-r+2} \right]$$

$$= \frac{(n+1)!}{(r-1)!(n-r+1)!} \times \left[\frac{n-r+2+r}{r(n-r+2)} \right]$$

$$= \frac{(n+1)!}{(r-1)!(n-r+1)!} \times \frac{(n+2)}{r(n-r+2)}$$

$$= \frac{(n+1)! \times (n+2)}{[(r-1)! \times r] [(n-r+1)! \times (n-r+2)]}$$

$$= \frac{(n+2)!}{r!(n+2-r)!} = {}^{n+2}C_r$$

Now , ${}^{n+2}P_r = \frac{(n+2)!}{(n+2-r)!}$

$$\therefore {}^{n+2}C_r = \frac{{}^{n+2}P_r}{r!}$$

$$\therefore {}^{n+1}C_r + {}^{n+1}C_{r-1} = \frac{{}^{n+2}P_r}{r!}$$

79 Answer :

[TOP](#)

[≤](#)

Reason : Let the following notations be used:

[TOP](#)
>

A : Engineer has a graduate level qualification

B: Engineer has a post-graduate level qualification

C: Engineer is below 30 years of age

D: Engineer is between 30 and 40 years of age

E: Engineer is above 40 years of age

$$(a) \quad (a) \quad P(A) = \frac{90}{120} = 0.75$$

$$(b) \quad P(B/E) = \frac{P(B \text{ and } E)}{P(E)} = \frac{\left(\frac{6}{120}\right)}{\left(\frac{30}{120}\right)} = \frac{6}{30} = 0.20$$

$$(c) \quad P(C/A) = \frac{P(C \text{ and } A)}{P(A)} = \frac{\left(\frac{54}{120}\right)}{\left(\frac{90}{120}\right)} = \frac{54}{90} = 0.60$$

$$(d) \quad (d) \quad P(D \text{ and } A) = \frac{12}{120} = 0.10$$

$$(e) \quad P(D/B) = \frac{P(D \text{ and } B)}{P(B)} = \frac{\left(\frac{18}{120}\right)}{\left(\frac{30}{120}\right)} = \frac{18}{30} = 0.60$$

[< TOP OF THE DOCUMENT >](#)